DRAG ON A SATELLITE MOVING ACROSS A SPHERICAL GALAXY
I: TIDAL AND FRICTIONAL FORCES IN SHORTLIVED ENCOUNTERS

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ABSTRACT

The drag force on a satellite of mass M moving with speed $\mathbf{V}$ in the gravitational field of a spherically symmetric background of stars is computed. During the encounter, the stars are subject to a time-dependent force that alters their equilibrium. The resulting distortion in the stellar density field acts back to produce a force $\mathbf{F}_\triangle$ that decelerates the satellite. This force is computed using a perturbative technique known as *linear response theory*.

In this paper, we extend the formalism of linear response to derive the correct expression for the back-reaction force $\mathbf{F}_\triangle$ that applies when the stellar system is described by an equilibrium *one-particle* distribution function. $\mathbf{F}_\triangle$ is expressed in terms of a suitable correlation function coupling the satellite dynamics to the unperturbed dynamics of the stars. At time t the force depends upon the whole history of the composite system. In the formalism, we account for the shift of the stellar center of mass resulting from linear momentum conservation. The self-gravity of the response is neglected since it contributes to a higher order in the perturbation. Linear response theory applies also to the case of a satellite orbiting outside the spherical galaxy.

The case of a satellite moving on a straight-line, at high speed relative to the stellar dispersion velocity, is explored. We find that the satellite during its passage rises (a) *global tides* in the stellar distribution and (b) a *wake*, i.e., an overdense region behind its trail.

If the satellite motion is external to the galaxy, it suffers a *dissipative* force which is not exclusively acting along $\mathbf{V}$ but acquires a component along $\mathbf{R}$, the position vector relative to the center of the spherical galaxy. We derive the analytical expression of the force, in the impulse approximation.

In penetrating shortlived encounters, the satellite moves across the stellar distribution and the wake excited in the density field is responsible for most of the deceleration. We find that *dynamical friction* rises from a *memory* effect involving only those stars perturbed along the path. The force can be written in terms of an effective Coulomb logarithm which now depends on the dynamical history, and in turn upon time t. $\ln\Lambda$ is computed for two simple equilibrium density distributions: It is shown that the drag increases as the the satellite approaches the denser regions of the stellar distribution. The braking force then stays almost constant. When the satellite crosses the edge of the galaxy the force does not vanish but declines with time as $\mathbf{R}\to\infty$.

In the case of a homogeneous cloud, we compute the total energy loss. In evaluating the contribution resulting from friction, we derive self-consistently the maximum impact parameter which is found equal to the length travelled by the satellite within the system. Tides excited by the satellite in the galaxy reduce the value of the energy loss by friction; in close encounters this value is decreased by a factor ~ 1.5.

Subject headings: galaxies: clustering – stars:stellar dynamics

I. INTRODUCTION

A massive object M moving with velocity $\mathbf{V}$ through a background of field stars suffers a dissipative force known as dynamical friction. In the original formulation of Chandrasekhar (1943), this force is the result of the momentum exchange between the particle M and the stars of the background. In a uniform

isotropic stellar system, the uncorrelated superposition of these binary encounters leads to a frictional force

$$\mathbf{F}_{DF} = -4\pi \frac{[GM]^2}{V^2} \rho(< V) \ln \left(\frac{b_{max}}{b_{min}} \right) \frac{\mathbf{V}}{V}$$

where ρ is the mass density of the stars having speed less than V, and b_{max} and b_{min} are the maximum and minimum impact parameters for which encounters can be considered effective. b_{max} is conventionally set equal to the size of the stellar system surrounding the object (to avoid a divergence) and b_{min} is the larger between the characteristic radius of the satellite M and the impact parameter for a 90^o scattering event (White 1976). In the derivation of the above equation the stars interact solely with the gravitational field of the incoming particle and move along trajectories that are straight lines at infinity. Because of the isotropy of the surrounding stellar field the force on M, responsible for its deceleration, is acting exclusively along the direction of motion, i.e., along $\mathbf{V}$. Dynamical friction can be viewed, alternatively, as a force originating from the overdensity excited in the stellar field by the particle in its motion: a wake develops behind M which is causing its deceleration (Mulder 1983).

Dynamical friction plays a central role in many astrophysical environments: It intervenes, for example, in the evolution of galaxies accreting small dwarfs (Lin & Tremaine 1983; Quinn & Goodman 1986; Binney & Tremaine 1987), in galaxy cannibalism, in the collision and merger of galaxies (Barnes & Hernquist 1992), and in the inspiral of a massive black hole at the center of dense stellar clusters (Begelman, Blandford & Rees 1980). The expression of the frictional force in a homogenous background has therefore been used to derive simple analytical estimates of the orbital decay times and to track evolutionary scenarios, for these cases. In these systems however, the ambient medium is neither uniform, nor infinite and this calls for a deep analysis of this process.

Since the work by Chandrasekhar, dynamical friction has been studied using various techniques. Tremaine and Weinberg (1984), and Weinberg (1986) studied the loss of angular momentum suffered by a satellite moving in the gravitational field of a spherical galaxy, along a circular orbit. Using a perturbative approach (that refers also to works by Lynden-Bell & Kalnajs 1972) for the description of the interaction of the satellite with the background of self-gravitating point masses, they found that angular momentum is exchanged secularly with those stars whose orbits are commensurate, i.e., in near-resonance with the satellite's orbit. In applying the technique to the case of a satellite (modelled as a Plummer sphere) orbiting inside of a singular isothermal sphere, Weinberg (1986) found that the resulting torque on M is equivalent in magnitude to the one obtained using Chandrasekhar formula, applied to a locally homogeneous system: The maximum impact parameter b_{max} is found to be comparable to the size of the satellite orbit R_s. This result suggests that the braking of a satellite depends more on the local properties of the stellar background, despite the long-range nature of the gravitational force.

For a parent galaxy modelled as a polytrope and a satellite as a non-deformable Plummer sphere similar findings were inferred by Bontekoe & Van Albada (1987). In a self-consistent N-body numerical calculation these authors observed that the decay process can still be described as if it were a local phenomenon in which the density field around the object determines primarily the magnitude of the torque and in turn the decay time, for a satellite moving along a circular orbit (see also Farouki, Duncan & Shapiro 1983). Bontekoe & Van Albada (1987) also explored the evolution of a satellite moving around the primary galaxy on a grazing orbit, and found that after a few revolutions the satellite, losing progressively energy and angular momentum, enters the stellar distribution and suffers complete merger. This case illustrates the inadequacy of Chandrasekhar formula since it would predict no drag for a satellite orbiting outside the outer edges of the companion galaxy. But not only, it clearly indicates that the satellite experiences a deceleration that rises in response to the perturbation that is excited in the stellar field during the interaction. This is a global phenomenon that can be explained in terms of an interaction with resonant stars in the galaxy (Tremaine & Weinberg 1984). Thus a continuity should exist in the physical processes inducing orbital decay.

The aim of this paper and of the accompanying one is to clarify the role of the tidal and frictional forces in affecting the motion of the satellite. This is achieved within the context of the theory of *linear response*. The method is close to that explored by Bekenstein and Maoz (1992) who relate the frictional drag to the stochastic components of the background forces, thus establishing a connection between dynamical friction and the fluctuation-dissipation theorem. Their method proved useful in giving a unified picture of the process of energy loss and heating of a test particle traveling through a uniform stellar system. In a

subsequent work, Maoz (1993) provided an approximate expression for the energy loss experienced by a massive particle moving in a nonhomogeneous medium. He noted that, owing to the nonuniformities present in the underlying stellar system, dynamical friction no longer depends on local background characteristics and that the force acquires a component in the direction of the spatial density gradients. Nevertheless he does not compute the force nor give a complete and consistent description of the process.

In this paper we compute the force of feed-back on a satellite moving in a background which is nonuniform and spherically symmetric, improving the perturbative technique first developed by Kandrup (1981,1983; see also the recent work by Nelson & Tremaine 1997). The collisionless system of stars is described by the equilibrium distribution function f_0. During the passage of the satellite M, the stars are subject to a time-dependent gravitational force and on account of that, the distribution function modifies: Its new value $f = f_0 + \Delta f$ is calculated using the theory of the linear response. The perturbation induced by the satellite acts back producing a force $\mathbf{F}_\Delta$ which is computed, accordingly, as expectation value on the response function Δf of the microphysical force resulting from the interaction of M with the stars of the background. The characteristics of this force are closely related to the way the stellar background responds to the perturbation and depend, as in memory effect, on the whole history of the particle and stars since the moment in which the interaction is turned on. The formalism also applies to the case of a satellite orbiting outside of the stellar distribution.

In this paper we wish to shed light into the nature of the force and derive a natural decomposition in terms of a global and local response of the background. This work is also preparatory to the analysis that will be carried in paper II, on the role of tides in causing the orbital decay of a satellite accreting onto a companion galaxy, a process relevant to the clustering of structures in cosmology.

In addressing all these problems, for ease of analysis, and consistently with the hypothesis that stars in equilibrium can be considered as a collisionless system, the equilibrium distribution function f_0 is factorized. This assumption is customarily used to compute mean properties of the response, as in Weinberg approach (1986). We here find that the introduction of such hypothesis cancels out artificially an important correlation present among the stars: Under the action of their self-gravity, the stars perform bound orbits around the barycenter of the galaxy. The displacement that the center of mass suffers during the encounter with the satellite (owing to linear momentum conservation) represents a collective response neglected in previous analysis. This response needs to be included for a correct estimate of the transfer of orbital energy into the internal degrees of freedom of the galaxy.

In this paper we modify the formalism of the theory of linear response to account for this important effect. This modification explains the longstanding problem of the differences in the sinking times found in N-body simulations (White 1983; Zaritsky & White 1988; Hernquist & Weinberg 1988) and in semi-restricted methods (Lin & Tremaine 1983). We present here the method used to include this effect but defer to paper II for a more thorough analysis.

The layout of the paper is as follows: In §2 we sketch the method used to compute the macroscopic force on M, using a derivation which is independent of that given by Kandrup (1981). In §3 we test the theory against known results, focusing on shortlived distant encounters. In §4 we derive the formal expression of the force $\mathbf{F}_\Delta$ on M that accounts for the shift of the center of mass of the galaxy. We then compute in §5, the force on the satellite M moving at high speed through a nonuniform stellar background characterized at equilibrium by a maxwellian distribution function. In §6 we compute the frictional force on a satellite moving across a finite size cloud with given density profile, and compute the shape of the tidal distortion. In §7 we evaluate the extent of the total energy loss. In §8 we present our conclusions.

II. THEORY OF LINEAR RESPONSE

2.1 *Response Function and Back-Reaction Force*

Consider a particle (i.e., a satellite) of mass M and velocity $\mathbf{V}$ moving through a collisionless system of N stars. The stars have mass $m \ll M$ and are described by an equilibrium distribution function f_0. At

$\mathbf{R}(t)$, the massive particle experiences an N-body force resulting from the gravitational interaction with the background stars

$$\mathbf{F}(t) = -GMm \sum_{i=1}^{N} \frac{\mathbf{R}(t) - \mathbf{r}_i}{|\mathbf{R}(t) - \mathbf{r}_i|^3} \tag{1}$$

where $\mathbf{r}_i$ is the position of the $i-$th star at time t. By action-reaction, each of the N stars of the background experiences a *perturbing* force

$$\mathbf{F}_i(t) = GMm \frac{\mathbf{R}(t) - \mathbf{r}_i}{|\mathbf{R}(t) - \mathbf{r}_i|^3}. \tag{2}$$

Under the influence of the time dependent force $\mathbf{F}_i$, the stellar distribution function f is altered and the distortion with respect to equilibrium can be described, to first order in the perturbation, in terms of the *response* function Δf such that

$$f = f_0 + \Delta f. \tag{3}$$

Because of the change in the distribution function induced by the satellite M, the force on M changes. This *macroscopic* force can be determined as mean on the distribution function f and is decomposed into two terms

$$< \mathbf{F} >_{f_0 + \Delta f} = < \mathbf{F} >_{f_0} + < \mathbf{F} >_{\Delta f} . \tag{4}$$

The force $< \mathbf{F} >_{f_0}$ originates from the interaction of M with the *smooth potential* generated by the *unperturbed* stellar system; the term $< \mathbf{F} >_{\Delta f}$ instead will represent the force of *back-reaction* whose magnitude depends on the response function Δf. As a first step, we calculate the perturbation Δf on the stellar distribution function.

Let $-\mathbf{F} = \sum_{i=1}^{N} \mathbf{F}_i$ be the applied time dependent force acting on the stellar background. The distribution function $f = f_0 + \Delta f$ can be computed, to first order in the perturbation, using the theory of *linear response* (TLR hereafter). For this purpose we need to specify the form of the disturbance and the properties of the stellar system in the *unperturbed* state through the equilibrium Hamiltonian H_0.

In the collisionless stellar system

$$H_0 = \sum_{i=1}^{N} \frac{p_i^2}{2m} \quad + \Psi. \tag{5}$$

where Ψ describes the smooth self-interaction, i.e. the mean field of the gravitating system. In presence of the time dependent external force induced by the satellite, the Hamiltonian modifies and the perturbation H_1 is simply

$$H_1(t) = -\sum_{i=1}^{N} \frac{GMm}{|\mathbf{R}(t) - \mathbf{r}_i|}. \tag{6}$$

H_1 results from the superposition of the gravitational force that the incoming body exerts on each particle of the system. H_1 is regarded as an explicit function of t depending on the instantaneous position $\mathbf{R}(t)$ of the particle M and as a function of the $6N$ coordinates $(\vec{r}, \vec{p})$ of the phase space: hence, $H_1 = H_1(t; \Gamma)$. (Γ denotes shortly the phase-space coordinates)

According to the Liouville equation, the evolution of the distribution function f is given by the equation

$$\frac{\partial f(t)}{\partial t} = \{H_0 + H_1(t), f(t)\}. \tag{7}$$

If the distribution function f_0 depends on the phase-space coordinate only through H_0 the Poisson bracket $\{H_0, f_0\}$ vanishes identically, and to first order in the perturbation, we can approximate equation (7) as

$$\frac{\partial \Delta f(t)}{\partial t} = \{H_0, \Delta f\} + \{H_1(t), f_0\}. \tag{8}$$

The formal solution is

$$\Delta f(t; \Gamma) = \int_{-\infty}^{t} ds \ \mathrm{e}^{-i(t-s)\mathcal{L}} \ \{H_1(s), f_0\}, \tag{9}$$

where $\mathcal{L} \equiv i\{H_0, \ \}$ denotes the Liouville operator acting on the $6N$ coordinates of the phase space. In equation (9) the expression of the Poisson bracket reads

$$\{H_1(s;\Gamma), f_0\} = -GMm \sum_{i=1}^{N} \frac{\mathbf{R}(s) - \mathbf{r}_i}{|\mathbf{R}(s) - \mathbf{r}_i|^3} \cdot \nabla_{\mathbf{p}_i} f_0. \tag{10}$$

In defining the limits of integration of equation (9), the external force is applied in the infinite past when the system was in a stationary state and the particle M far away: accordingly, H_1 vanishes for $t \to -\infty$.

Having calculated the distortion Δf, (to first order in the perturbation H_1) we can compute the expectation value of the microphysical force $\mathbf{F}$ acting on the satellite M at time t :

$$< \mathbf{F}(t) >_{\Delta f} \equiv \int d\Gamma \ \Delta f \ \mathbf{F}(t;\Gamma). \tag{11}$$

For a perturbation of the form given by equation (10)

$$< \mathbf{F}(t) >_{\Delta f} = -GMm \sum_{i=1}^{N} \int_{-\infty}^{t} ds \int d\Gamma \ e^{is\mathcal{L}} \left[\nabla_{\mathbf{p}_i} f_0 \cdot \frac{\mathbf{R}(s) - \mathbf{r}_i}{|\mathbf{R}(s) - \mathbf{r}_i|^3} \right] \ e^{it\mathcal{L}} \left[\mathbf{F}(t;\Gamma) \right] \tag{12}$$

where we used the property that $\mathcal{L}$ is a self-adjoint operator. In equation (12) the Liouville operator acts on the phase-space coordinates (Γ): applied to a generic dynamical variable A (representing any component of the phase-space coordinates which satisfies equation $\partial A/\partial t = i\mathcal{L}A$) $\mathcal{L}$ carries A along its evolution from time $t = 0$ to time t , i.e.,

$$A(\Gamma(t)) = \mathrm{e}^{it\mathcal{L}} A(\Gamma(0)) \tag{13}$$

Hence, we interpret $\mathrm{e}^{it\mathcal{L}}\mathbf{F}$ at an arbitrary time t as the value of the force $\mathbf{F}$ acting on M at $\mathbf{R}(t)$ resulting from the gravitational interaction with the particles that evolved from $t = 0$ to time t along a path whose dynamics is uniquely determined by the unperturbed Hamiltonian H_0. Likewise, $e^{is\mathcal{L}}$ acts on the scalar product to yield accordingly

$$< \mathbf{F}(t) >_{\Delta f} = -GMm \sum_{i=1}^{N} \int_{-\infty}^{t} ds \ \int \ d\Gamma \ \left[\nabla_{\mathbf{p}_i(s)} f_0 \ \cdot \frac{\mathbf{R}(s) - \mathbf{r}_i(s)}{|\mathbf{R}(s) - \mathbf{r}_i(s)|^3} \right] \ \mathbf{F}(t;\Gamma(t)) \tag{14}$$

We will shortly denote $< \mathbf{F}(t) >_{\Delta f} = \mathbf{F}_\Delta$ hereafter. This equation provides the formal expression of the force acting on M in terms of the microscopic force $\mathbf{F}$ and of the equilibrium distribution function f_0. We notice that within TLR, the response function Δf is of order "G"; Hence, the resulting force describing the feed back mechanism acting on M is of order "G^2". Equation (14) contains all details of the microphysical interaction of the N stars with the perturber: It accounts for the *self-gravity* of the stellar equilibrium and for the actual dynamics of the perturber.

The difficulty in evaluating the multiple integrals of equation (14), involving the actual dynamics of the underlying stars, is enormous particularly if we want to use analytic tools to gain insight on the mechanism producing friction. To overcome this problem we introduce a major simplification knowing that the stellar motion in a galaxy can be described in terms of a mean field potential (Binney & Tremaine 1987): The system can therefore be regarded as collisionless and stars behave as independent particles. The corresponding distribution function $f_0(\Gamma)$ can therefore be written in terms of the one-particle phase space density f^{op}:

$$f_0 = \Pi_{i=1}^{N} f^{\mathrm{op}}(\mathbf{r}, \mathbf{p}). \tag{15}$$

Under this hypothesis, due to the statistical independence of the particles, the cross-correlation terms (i.e., those involving different indices in the summation of eq.[14]), cancel identically, in the limit $N \gg 1$. Only the term of self-correlation survives giving rise to a macroscopic force

$$\mathbf{F}_\Delta = [GM]^2 N m^2 \int_{-\infty}^{t} ds \int d_3\mathbf{r} \, d_3\mathbf{p} \ \left[\nabla_{\mathbf{p}(s)} \ f^{\mathrm{op}} \cdot \frac{\mathbf{R}(s) - \mathbf{r}(s)}{|\mathbf{R}(s) - \mathbf{r}(s)|^3} \right] \ \frac{\mathbf{R}(t) - \mathbf{r}(t)}{|\mathbf{R}(t) - \mathbf{r}(t)|^3} \tag{16a}$$

where the phase-space volume $d_3\mathbf{r}\, d_3\mathbf{p}$ and $f^{\rm op}$ are independent of time and can be referred to time t. An equivalent expression for the force can be obtained by applying the evolution operator $e^{-i(t-s)\mathcal{L}}$ to $\mathbf{F}(t;\Gamma)$ in equation (12) and reads

$$\mathbf{F}_\Delta = [GM]^2 N m^2 \int_{-\infty}^{t} ds \int d_3\mathbf{r}\, d_3\mathbf{p}\ \left[\nabla_\mathbf{p}\ f^{\rm op} \cdot \frac{\mathbf{R}(s)-\mathbf{r}}{|\mathbf{R}(s)-\mathbf{r}|^3}\right]\ \frac{\mathbf{R}(t)-\mathbf{r}(t-s)}{|\mathbf{R}(t)-\mathbf{r}(t-s)|^3} \tag{16b}$$

(equation (16b) will be used in paper II).

Introducing the velocity vector $\mathbf{v} = \mathbf{p}/m$, we can express $f^{\rm op}$ as a function of $(\mathbf{r}, \mathbf{v})$. If the equilibrium system is described by a Gaussian distribution function

$$f^{\rm op}(\mathbf{r}, \mathbf{v}) = n_0(r) \left(\frac{\beta m}{2\pi}\right)^{3/2} \exp(-\frac{1}{2}\beta m v^2) \tag{17}$$

we have $\nabla_\mathbf{v} = -\beta f^{\rm op}\ \mathbf{v}$. In equation (17), r and v denote the absolute values of $\mathbf{r}$ and $\mathbf{v}$ and the coefficient $\beta \equiv (m\sigma^2)^{-1}$ is directly related to the one-dimensional dispersion velocity σ characteristic of the equilibrium stellar distribution; $n_0(r)$ is the unperturbed probability density which for a spherically symmetric system is a function of r only. $f^{\rm op}$ is defined so that $\int d_3\mathbf{r}\ n_0(r) = 1$. With the change of variables $(\mathbf{r}, \mathbf{p}) \to (\mathbf{r}, \mathbf{v})$ and according to the normalization used we recall that $d_3\mathbf{r}\, d_3\mathbf{p} f^{\rm op}(\mathbf{r}, \mathbf{p})$ is equal to $d_3\mathbf{r}\, d_3\mathbf{v} f^{\rm op}(\mathbf{r}, \mathbf{v})$.

The force $\mathbf{F}_\Delta$ at time t is found to depend on the whole " history " of the composite system, i.e., on the dynamics of the satellite, and on the dynamics of stars of the background as determined by the (mean field) Hamiltonian H_0. If we introduce the tensor

$$T^{ba} \equiv -[GM]^2 N m^2 \beta \frac{R^b(s) - r^b(s)}{|\mathbf{R}(s)-\mathbf{r}(s)|^3} \frac{R^a(t) - r^a(t)}{|\mathbf{R}(t)-\mathbf{r}(t)|^3} \tag{18}$$

correlating the microscopical force at time s with that at time t, we can interpret equation (16) written in the form

$$F^a_\Delta = \int_{-\infty}^{t} ds\ \int d_3\mathbf{r}\ d_3\mathbf{v}\ f^{\rm op} v^b(s) T^{ba} \tag{19}$$

as a manifestation on the *fluctuation-dissipation theorem* relating the averaged force of back-reaction to the time integral of a suitable dynamical correlation function of the unperturbed system (Bekenstein & Maoz 1992; Nelson & Tremaine 1997). Due to the long range nature of the gravitational interaction all stars in the galaxy give a sizable contribution to the force. $\mathbf{F}_\Delta$ is a force which is intrinsically nonlocal both in phase-space and time. In equation (19), $v^b\, T^{ba}$ are shorthand for the summations (hereafter).

Notice that in the steps that lead to equation (19) the perturber's motion needs not necessarely to be confined within the stellar distribution. We can therefore evaluate, within TLR, the force on the massive particle M when it is orbiting outside the galaxy.

2.2 *Force and Density Distortion*

$\mathbf{F}_\Delta$ can be alternatively viewed as a force rising in response to the distortion caused by the satellite on the stellar density field. The satellite M induces a disturbance in the density distribution function $n(\mathbf{r})$ that can be readily evaluated, within TLR, from equation (9). Relative to its unperturbed value n_0, the density changes by an amount equal to

$$\Delta n(\mathbf{r}) = \int d_3\mathbf{r}''\ d_3\mathbf{v}\ \delta_3(\mathbf{r}'' - \mathbf{r}) \Delta f(\mathbf{r}'', \mathbf{v}) \tag{20}$$

where δ_3 denotes the 3-dimensional Dirac function. From the solution (9) and equation (10) we have

$$\Delta n(\mathbf{r}) = -GM \int_{-\infty}^{t} ds\ \int d_3\mathbf{r}''\, d_3\mathbf{v} \delta_3(\mathbf{r}''(t-s) - \mathbf{r}) \nabla_\mathbf{v} f^{\rm op}(\mathbf{r}'', \mathbf{v}) \cdot \frac{\mathbf{R}(s) - \mathbf{r}''}{|\mathbf{R}(s) - \mathbf{r}''|^3}. \tag{21}$$

In the steps that lead to equation (21) we have used explicitly the self-adjointness property of the Liouville operator and the factorization of f_0 ($\mathcal{L}$ is applied to the phase-space coordinates of the Dirac function). Under the disturbance (21), the satellite experiences a force

$$\mathbf{F}_\Delta = -GMNm \int d_3\mathbf{r}\Delta n(\mathbf{r}) \frac{\mathbf{R}(t) - \mathbf{r}}{|\mathbf{R}(t) - \mathbf{r}|^3} \tag{22}$$

which coincides with equation (16b). The expression of the force $\mathbf{F}_\Delta$ will be modified in §4 to account properly for the shift of the centre of mass of the stellar distribution, that occurs during the gravitational interaction.

Given $\mathbf{F}_\Delta$, we can formally compute the total energy loss suffered by the satellite

$$\Delta E = \int_{-\infty}^{+\infty} dt\ \mathbf{F}_\Delta \cdot \mathbf{V} = -\frac{1}{2}[GMm]^2 N\beta \int d_3\mathbf{r}\, d_3\mathbf{v}\ f^{\mathrm{op}}\mathcal{J}^2 \tag{23}$$

where

$$\mathcal{J} = \int_{-\infty}^{+\infty} dt\ \mathbf{v} \cdot \frac{\mathbf{R} - \mathbf{r}}{|\mathbf{R} - \mathbf{r}|^3}. \tag{24}$$

Hereafter $\mathbf{R}$, $\mathbf{r}$, and $\mathbf{v}$ will denote the vectors at the current time t and the volume element $d_3\mathbf{r}\, d_3\mathbf{v}$ can be referred only for convenience to time t. (The derivation of equation (23) is summarized in Appendix A.)

III. TESTING THE THEORY

Kandrup (1983) verified, in his early work, that equation (16) reproduces the expression of the drag force derived by Chandrasekhar for the case of a point mass moving through an infinite homogeneous stellar background. This important limit is recovered within TLR in the hypothesis that the stars and the perturber move along straight lines (we do not sketch here the derivation of this limit but refer to Kandrup). According to the original formulation (Chandrasekhar 1943), the gravitational interaction among the stars was neglected and the braking force on M was derived considering the momentum exchange resulting from the incoherent superposition of the binary encounters with single stars moving along hyperbolic orbits relative to M. TLR reproduces the correct results to order G^2, giving in the high speed limit, i.e, for $V \gg \sigma$ a frictional force

$$\mathbf{F}_{DF} = -4\pi \frac{[GM]^2}{V^2} Nmn_0 \ln\left(\frac{b_{max}}{b_{min}}\right) \frac{\mathbf{V}}{V} \tag{25}$$

where b_{max} is set equal to the characteristic length of the stellar system to avoid a “infrared ” divergence. In TLR, the above expression is derived introducing also at short distances a “ultraviolet” cutoff, b_{min}, the minimum impact parameter for collisions at large scattering angle. The presence of a divergence at small relative distances between M and the stars, in our formulation, is a consequence of the approach used. Since we have treated H_1 as a small perturbation, equation (9) becomes invalid when the distortion in the background produces a wake with overdensity $\Delta n/n_0 \sim 1$. The density enhancement $\Delta n/n_0$ (computed using equation (9) and neglecting the self-gravity of stars)

$$\frac{\Delta n}{n_0} = \frac{GM}{\sigma^2} \frac{1}{|\mathbf{R} - \mathbf{r}|} e^{(\alpha^2 - x^2)} \left[\mathrm{erf}\alpha + 1\right] \tag{26}$$

(with $\alpha \equiv (\mathbf{V}/\sqrt{2}\sigma) \cdot (\mathbf{R} - \mathbf{r})/|\mathbf{R} - \mathbf{r}|$, and $x \equiv V/\sqrt{2}\sigma$) becomes close to unity when the relative distance between a star and the satellite M is

$$\epsilon \sim \frac{GM}{\mathrm{Max}(\sigma^2, V^2)} \tag{27}$$

One can recognize that at such a relative distance the mean squared velocity of a typical field particle would increase by an amount of order σ (V, in the high speed limit). For an extended (nondeformable) satellite ϵ is instead set equal to physical extension of M. Thus ϵ gives the "permitted" size the satellite (White 1976). In the expression of the force $\mathbf{F}_\Delta$ the domain of integration is thus limited to $|\mathbf{R}-\mathbf{r}| \gg \epsilon$.

Can the theory of LR be tested against other known results ? The energy loss of an object of mass M moving at high speed V relative to a galaxy characterized by a mean-square radius $< r^2 >$ is known to be

$$\Delta E = -\frac{4}{3} Nm < r^2 > \frac{[GM]^2}{V^2\ b^4}. \tag{28}$$

This equation is derived in the hypothesis that the distance of closest approach b largely exceeds the radius of the disturbed galaxy and that the effective duration of the encounter τ_e be shorter than the crossing (dynamical) time of the stars $\tau_{dyn} \sim (< r^2 > / < v^2 >)^{1/2}$ where $< v^2 >$ is the mean squared velocity ($< v^2 >= 3\sigma^2$). This limit is known as impulse approximation and applies when the perturber moves at high speed ($V \gg \sigma$) relative to the stars. The energy loss (28) is of order "G^2"and was derived by Spitzer (1958) on the basis of simple arguments, without direct knowledge of the force causing such a loss (Binney & tremaine 1987).

Using equation (23) we recover Spitzer's results as in high speed encounters the energy loss of the satellite is exceedingly small (scaling with V^{-2}), and TLR applies. The motion of the perturber can be approximated as linear while stars barely move since the interaction is shortlived. Under these simplifications, equation (23) for the energy loss of M can be evaluated analytically (the calculation is sketched in Appendix A) to give

$$\Delta E = -2Nm\frac{[GM]^2}{V^2\ b^2} - \frac{4}{3} Nm < r^2 > \frac{[GM]^2}{V^2\ b^4} + O\left(\frac{< r^2 >}{b^2}\right) \tag{29}$$

ΔE as expected is the sum of two contributions. The first term, according to linear momentum conservation, is due to *recoil*, i.e., the effect representing the exchange of energy between M and the galaxy viewed as pointlike. The second instead describes the transfer of orbital energy into the internal energy of the stellar distribution.

IV. TLR REVISED: THE FORCE IN THE GALAXY REST FRAME

4.1 *Correlation and shift of stellar barycenter*

In this paper, we wish to gain insight into the mechanisms that ultimately cause the transfer of orbital energy into the internal degrees of freedom of the stellar distribution. For this purpose, we need to isolate from the force $\mathbf{F}_\Delta$ those terms responsible to the recoil, representing only a global displacement of the collection of stars in the system with no relation to the "binding" mechanism that M experiences as a result of friction. We note that equation (16) derived under the hypothesis that stars, in equilibrium, behave as *independent particles*, does not correctly describe the energy transfer during the interaction. At the basis of this inconsistency is the simplifying assumption of factorization of f, the equilibrium distribution function (eq. [15]). According to this hypothesis, the dynamics of each star is uncorrelated from the dynamics of any other star in the galaxy. In this approximation each star moves in the averaged potential generated by all the other stars. For a spherical distribution this potential gives rise to a central force binding the stars to the center of mass of the galaxy. In the encounter with a satellite, the center of symmetry of the stellar distribution suffers a rigid displacement due to linear momentum conservation. The stars thus coherently move responding to the satellite perturbation with a coordinated shift of their orbits. However, this correlation (present in the full formalism leading to eq. [14]) is lost, since the mean field dynamics we use to represent the unperturbed motion of the stars (eq. [15]) does not allow for the shift of the center of mass; the dynamics has lost its *translational invariance*.

As a consequence, equation (23) for ΔE gives a consistent estimate of the energy loss only in three limiting cases; when (i) the satellite moves through a homogeneous infinite background of non-interacting

stars, (ii) when the interaction is shortlived, as in the impulse approximation or (iii) when the center of symmetry of the galaxy is *pinned*. In the encounter of a satellite with a spherical galaxy this last hypothesis is clearly unphysical. The above considerations rise therefore a problem of consistency which can not be overlooked.

To solve for this difficulty, we examine the process of dynamical friction not from the laboratory system but from the noninertial frame of reference comoving with the center of mass of the perturbed galaxy: $\mathbf{X} = (1/N) \sum_i \mathbf{r}_i$. Newton's equations, in this frame, read

$$\mu \frac{d^2(\mathbf{R}-\mathbf{X})}{dt^2} = < \mathbf{F} >_{f^{\rm op}} + \mathbf{F}_\Delta \tag{30}$$

$$m \frac{d^2(\mathbf{r}_i - \mathbf{X})}{dt^2} = \mathbf{F}_{internal} + (-\mathbf{F}_i) - \frac{1}{N}\sum_{j=1}^{N}(-\mathbf{F}_j). \tag{31}$$

The satellite simply acquires a new mass equal to the reduced mass of the system $\mu = MNm/(M+Nm)$. Instead, the rhs of equation (31) for the motion of the $i-$th star acquires a new term, representing the opposite of the "mean" force exerted by the satellite on the galaxy, as a whole ($\mathbf{F}_i$ is the force acting on M due to the $i-$th star; eq. [2]).

Guided by equations (30) and (31), we are led to modify $\mathbf{F}_\Delta$ introducing a new correlation tensor T obtained subtracting such a " mean" force to the kernel $[\mathbf{R}-\mathbf{r}]/|\mathbf{R}-\mathbf{r}|^3$ along the complete history until time t, yielding

$$\begin{aligned}\mathbf{F}_\Delta = &- [GM]^2 N m^2 \beta \int_{-\infty}^{t} ds \int d_3\mathbf{r}\, d_3\mathbf{v}\, f^{\rm op} \\ &\left\{ \mathbf{v}(s) \cdot \left[\frac{\mathbf{R}(s)-\mathbf{r}(s)}{|\mathbf{R}(s)-\mathbf{r}(s)|^3} - \int d_3\mathbf{r}' n_0(r') \frac{\mathbf{R}(s)-\mathbf{r}'}{|\mathbf{R}(s)-\mathbf{r}'|^3} \right] \right\} \frac{\mathbf{R}-\mathbf{r}}{|\mathbf{R}-\mathbf{r}|^3}.\end{aligned} \tag{32}$$

$\mathbf{F}_\Delta$ represents here the response force acting of M at time t as measured in the frame comoving with the perturbed stellar distribution. We now regard $\mathbf{R}$ and $\mathbf{r}$ as coordinates relative to the center of mass of the perturbed stellar system.

In this frame, the disturbance in the density field is calculated according to equation (32), by introducing the modified kernel. For a Gaussian distribution function, we have

$$\begin{aligned}\Delta n(\mathbf{r}) = &GMm\beta \left(\frac{m\beta}{2\pi}\right)^{3/2} \int ds \int d_3\mathbf{r}''\, d_3\mathbf{v}\ \delta_3(\mathbf{r}''(t-s)-\mathbf{r}) n_0(r'') e^{-[\frac{1}{2}\beta m v^2]} \\ &\mathbf{v} \cdot \left[\frac{\mathbf{R}(s)-\mathbf{r}''}{|\mathbf{R}(s)-\mathbf{r}''|^3} - \int d_3\mathbf{r}'\ n_0(r') \frac{\mathbf{R}(s)-\mathbf{r}'}{|\mathbf{R}(s)-\mathbf{r}'|^3} \right]\end{aligned} \tag{33}$$

with $\mathbf{r}'$ a dummy variable. Equation (32) (or equivalently eqs. [33] and [22]) provides the *correct* expression for the force $\mathbf{F}_\Delta$ on the satellite moving in the gravitational field of the primary galaxy.

4.2 *Self-gravity of the response*

The shift of the stellar barycenter was viewed by Hernquist and Weinberg (1989) as a manifestation of the self-gravity of the response. According to our analysis, the modified kernel in equation (32) derived to account for this displacement is introduced to restore the correlation cancelled in the formalism of TLR when $f^{\rm op}$ is introduced to represent the equilibrium stellar system. This is a key correction that accounts for the *self-gravity of the stellar background.* We underline the fact that in TLR the distribution function is perturbed only by the satellite *external* potential. The *self gravity of the response* has a different origin (White

1983; Nelson & Tremaine 1997): It expresses the “change” in the galaxy’s self-interaction potential induced by the perturbed density field (excited by the intruder). This would result in an additional component to the force $\mathbf{F}_{\Delta}$ on the satellite which is of higher order, and for this reason is neglected in our scheme. In fact, the response function Δf modified to account for the perturbed self-gravity would be of order G^2, leading to correction of order G^3 on the drag force.

This considerations are consistent with results, from numerical simulations by Bontekoe & Van Albada (1987) and Zaritsky & White (1988), on the negligible role played by the perturbed self-gravity in affecting the process of energy and angular momentum transfer by the satellite. The self-gravity of the equilibrium system is included and affects the process of energy transfer (as it will be shown in §6.5).

V. FRICTION AND TIDAL FORCE DECOMPOSITION

5.1 *High speed encounters*

In this Section we examine the case of a satellite M moving at high speed relative to a spherically symmetric stellar background. In this limiting case and to order “G^2”, the dynamics of the satellite and that of the stars can be approximated as

$$\mathbf{r}(s) = \mathbf{r} - (t-s)\mathbf{v} \tag{34}$$

$$\mathbf{R}(s) = \mathbf{R} - (t-s)\mathbf{V}, \tag{35}$$

with $\mathbf{V}$ and $\mathbf{v}$ constant vectors. The satellite and the stars thus move along straight lines, and we neglect the self-gravity of the stellar equilibrium system. The validity of this approximation thus restricts to encounters whose duration does not exceed the crossing time of the stars in the galaxy. In the high speed limit, the density disturbance thus reads

$$\begin{aligned}\Delta n(\mathbf{r}) =& GMm\beta\left(\frac{m\beta}{2\pi}\right)^{3/2}\int_{-\infty}^{t} ds \int d_3\mathbf{v}\; n_0(|\mathbf{r}-(t-s)\mathbf{v}|)e^{[-\frac{1}{2}\beta m v^2]}\\ &\mathbf{v}\cdot\left[\frac{\mathbf{R}-(t-s)\mathbf{V}-\mathbf{r}+(t-s)\mathbf{v}}{|\mathbf{R}-(t-s)\mathbf{V}-\mathbf{r}+(t-s)\mathbf{v}|^3} - \int d_3\mathbf{r}'\; n_0(r')\frac{\mathbf{R}-(t-s)\mathbf{V}-\mathbf{r}'}{|\mathbf{R}-(t-s)\mathbf{V}-\mathbf{r}'|^3}\right]\end{aligned} \tag{36}$$

as the presence of the Dirac function $\delta_3(\mathbf{r}''(t-s)-\mathbf{r})$ in the coordinate space relates the density at $\mathbf{r}$ to the density at $[\mathbf{r}-(t-s)\mathbf{v}]$. The spherical symmetry is lost since the satellite produces a distortion that keeps memory of the star and satellite motions; as a consequence, Δn is a function of vector $\mathbf{r}$.

The density response, when $V \gg \sigma$, should not depend on σ explicitly and its expression can be estimated in the limit $\beta \to \infty$. The relevant contributions to Δn arise, in this limit, from those stars having very low velocities for which βv^2 is finite. Hence, to $O(1/V^2)$, the density response Δn can be evaluated expanding the arguments of the integrals to first order in the velocity $\mathbf{v}$ (the calculation is sketched in Appendix B).

We find that the response function $\Delta n(\mathbf{r})$ naturally decomposes into two contributions: the first representing the wake rised by the satellite along his trail: The stream of particles behind M is characterized by an overdensity

$$\Delta_1 n(\mathbf{r}) = 4\pi GMn_0(r)\int_0^{\infty} d\tau\;\tau\delta_3(\mathbf{R}-\mathbf{r}-\tau\mathbf{V}). \tag{37}$$

The second contribution rises from the density inhomogeneities of the underlying unperturbed stellar system. Introducing the unit vector $\mathbf{n} \equiv \mathbf{V}/V$, we have

$$\Delta_2 n(\mathbf{r}) = -\frac{GM}{V^2}\mathbf{D}_{(\mathbf{R}-\mathbf{r})}\cdot\nabla_{\mathbf{r}} n_0(r) \tag{38}$$

with

$$\mathbf{D_Y} = \mathbf{S} + \mathbf{n}\left[\ln\frac{(|\mathbf{Y}|) - \mathbf{Y}\cdot\mathbf{n}}{R - \mathbf{R}\cdot\mathbf{n}}\right] + \frac{\mathbf{Y} - \mathbf{n}(\mathbf{Y}\cdot\mathbf{n})}{(|\mathbf{Y}|) - \mathbf{Y}\cdot\mathbf{n}} - \frac{\mathbf{R} - \mathbf{n}(\mathbf{R}\cdot\mathbf{n})}{R - \mathbf{R}\cdot\mathbf{n}}. \tag{39}$$

The distortion vector $\mathbf{D}$ gives the description of the global tides excited by the satellite during the encounter. In equation (39) the vector $\mathbf{S}$ is defined as

$$\mathbf{S}(\mathbf{R}) = \int_0^{+\infty} dx\, x \frac{\mathbf{R} - \mathbf{n}x}{|\mathbf{R} - \mathbf{n}x|^3}\bar{\Omega}(|\mathbf{R} - \mathbf{n}x|) \tag{40}$$

and

$$\bar{\Omega}(r) \equiv \int_{r'>r} d_3\mathbf{r}' n_0(r') \tag{41}$$

We have verified that the center of mass of the perturbed stellar system is not affected by the density distortion $\Delta n = \Delta_1 n + \Delta_2 n$. The displacement vector $\mathbf{S}$ compensates for the shift of the center of mass caused by the global tide, in the frame comoving with the galaxy. The result illustrates the importance of defining the appropriate physical frame of reference for the description of the gravitational interaction between the satellite and the primary stellar system.

Given the density distortion Δn, we can calculate the force acting on the satellite. We find that $\mathbf{F}_\Delta$ results in the superposition of two components., $\mathbf{F}_\Delta = \mathbf{F}_{DF} + \mathbf{F}_T$, the first representing the frictional drag induced by the density enhancement $\Delta_1 n$ that originates behind M

$$\mathbf{F}_{DF} = -\mathbf{n}\, 4\pi \frac{[GM]^2}{V^2}\int_{\epsilon/V}^{\infty} d\tau \frac{\rho_0(|\mathbf{R} - \tau\mathbf{V}|)}{\tau} \tag{42}$$

where the stellar mass density is defined as $\rho_0(r) = Nmn_0(r)$. In the expression of the force, the Coulomb logarithm (eq. [25]) is replaced by an integral, i.e., an "effective" $\ln\Lambda$ which depends on the density distribution along the wake, since the vector $\mathbf{R} - \tau\mathbf{V}$ selects only those stars streaming behind the satellite in its motion. We thus find that *dynamical friction rises from a memory effect involving only those stars perturbed along the motion of the satellite*. We may consider, loosely speaking, this contribution to the force as *local* since the drag on M is induced by the bow shock that the satellite excites in its supersonic motion across the medium (see eq. [26]). Equation (42) reproduces Chandrasekhar formula for an infinite homogeneous medium.

In addition to the frictional force, we discovered a new component that is closely related to the *tidal distortion* induced by the satellite in its passage. To first order in the perturbation, this component is

$$\mathbf{F}_T = \frac{[GM]^2}{V^2}\int d_3\mathbf{r}\ \mathbf{D}_{(\mathbf{R}-\mathbf{r})}\cdot\nabla_\mathbf{r}\rho_0(r)\ \frac{\mathbf{R} - \mathbf{r}}{|\mathbf{R} - \mathbf{r}|^3}. \tag{43}$$

The force $\mathbf{F}_T$ involves a volume integral (contrary to $\mathbf{F}_{DF}$) depending on the density gradients of the unperturbed stellar system: It therefore accounts for the *nonlocal* response of the galaxy to the disturbance and can be viewed as a force resulting from the tidal deformation rised by the satellite during the gravitational interaction. We will refer to as *tidal force* thereafter. Because of the nonhomogeneity of the underlying stellar background, $\mathbf{F}_T$ is a force acting not exclusively along the direction of motion: In a spherical system it acquires a component along $\mathbf{R}$, i.e., along the vector joining the center of the galaxy to the instantaneous position of the satellite. Notice that the decomposition of the force in terms of a tidal and frictional drag has been possible since we have described the interaction of the spherical system from the noninertial frame of reference comoving with the center of mass of the galaxy: The galaxy, as viewed in the laboratory frame, suffers in addition the recoil.

VI. DYNAMICAL FRICTION IN FINITE SIZE CLOUDS

6.1. *Uniform clouds*

As a first application, we estimate the frictional force experienced by a satellite traveling through a homogeneous spherical cloud of radius L. Consistently with equation (35), the satellite motion is approximated as uniform and introducing the impact parameter $\mathbf{b}$ defined so that $\mathbf{b} \cdot \mathbf{V} = 0$ we have

$$\mathbf{R}(t) = \mathbf{b} + t\mathbf{V}. \tag{44}$$

If $\mathbf{V}$ lies along the x-axis, and ψ denotes the angle between $\mathbf{V}$ and $\mathbf{R}$ (varying in between $(0, \pi)$), the satellite is seen to enter (exit) the stellar distribution at an angle ψ_1 (ψ_2) for which

$$\cot \psi_1 = -\frac{1}{b}(L^2 - b^2)^{1/2} \tag{45}$$

$$\cot \psi_2 = +\frac{1}{b}(L^2 - b^2)^{1/2} \tag{46}$$

In this case, the logarithmic integral that appears in equation (42) is readily estimated. When the satellite moves *inside* the cloud (i.e., when $\psi_1 < \psi < \psi_2$) it experiences a frictional force

$$\mathbf{F}_{DF} = -\mathbf{n}\; 4\pi \frac{[GM]^2}{V^2} \rho_0 \, \ln \Lambda_{\rm in} \tag{47}$$

where

$$\ln \Lambda_{\rm in} = \ln \frac{b \cot \psi + (L^2 - b^2)^{1/2} + \epsilon}{\epsilon}. \tag{48}$$

In Figure 1, the effective Coulomb logarithm ($\ln \Lambda$; solid line) is plotted against ψ for two selected values of b/L and for a cusp $\epsilon/L = GM/LV^2 = 0.01$. We notice that the frictional force, and in turn $\ln \Lambda$ increases in magnitude as the massive particle progresses along its motion inside the cloud: Early in the encounter the force is small but as the wake develops the drag rises. For sufficiently small (large) impact parameters relative to L, the force stays constant (steadily increases) until the edge of the cloud is reached. For $b \ll L$ the Coulomb logarithm becomes nearly independent of b and $b_{max} \to L$, the size of the cloud.

At angles $\psi < \psi_2$ the satellite finds itself outside of the stellar distribution. Despite that, the drag force is non vanishing because the stream of particles excited during its passage still produces a deceleration. This is a consequence of the memory effect present in the correlation tensor. The contribution to the force is given by

$$\mathbf{F}_{DF} = -\mathbf{n}\; 4\pi \frac{[GM]^2}{V^2} \rho_0 \, \ln \frac{b \cot \psi + (L^2 - b^2)^{1/2} + \epsilon}{b \cot \psi - (L^2 - b^2)^{1/2} + \epsilon} \tag{49}$$

with $\psi \le \psi_2$. For $\psi \to 0$, i.e., for $t \to \infty$, the logarithm slowly vanishes as $(2/b)(L^2 - b^2)^{1/2} \tan \psi \propto 1/t$.

In a real system, the force will decay as soon as the stars return to equilibrium, after a time $t \sim 1/\omega$, where with ω we denote the mean internal frequency ($\omega =< v^2 >^{1/2} / < r^2 >^{1/2}$). In the high speed limit, stars respond to the perturbation induced by the satellite as dust particles. Hence equation (49) is valid as long as the stellar dynamical time exceeds the time scale of the encounter τ_e, or equivalently $\omega R/V \ll 1$.

Notice that in addition to the frictional force $\mathbf{F}_{DF}$, TLR predicts also a tidal contribution that we will evaluate in §6.3.

6.2 *Nonuniform spherical galaxy*

In a spherical system with unperturbed density profile

$$\rho_0(r) = \rho_0^* \left[1 + \left(\frac{r}{L} \right)^2 \right]^{-\gamma} \tag{50}$$

the satellite approaches initially regions of increasing density and consequently suffers a drag whose extent with time (i.e., with decreasing ψ). At $\psi < \pi/2$ the medium begins to rarefy and the braking force weakens as the wake develops in a medium of decreasing density.

The magnitude of the force as a function of the orbital phase ψ can be computed analytically from equation (42) for $\gamma = 3/2$, defining the "effective" Coulomb logarithm as

$$\ln \Lambda = \int_{\epsilon/V}^{\infty} \frac{d\tau}{\tau} \left[1 + \left(\frac{|\mathbf{R} - \tau \mathbf{V}|}{L} \right)^2 \right]^{-\gamma}. \tag{51}$$

In this model, the mass within radius r declines logarithmically and the satellite in its motion remains always confined within the star's background. Figure 2 shows $\ln \Lambda$ for two values of the impact parameter $b = 0.9$ and 0.5, (respectively) expressed in units of L, the radial scale below which the density profile is almost uniform. We find that if the satellite moves in the halo of declining density only grazing the uniform density core, the drag reduces (by a factor of ~ 2 for $b = 0.9$) relative to the case of a cloud of radius L, when evaluated at phase $\psi \sim \pi/2$. Instead, at small impact parameters ($b \sim 0.1$) the Coulomb logarithm becomes comparable to that of a cloud of uniform density ρ_0^* and size L (Figure 1, solid line).

6.3 *Tidal Distortion*

Since the vector $(GM/V^2)\mathbf{D}$ describes the distortion in the density field induced by the satellite during its motion we can draw the isodensity contours of the perturbed stellar system. The first series of plots describe the deformation of a density contour lying inside R; the second reproduces the deformation induced by the passage of the perturber moving within the isodensity contour in the case of a uniform finite size cloud.

Figure 3 illustrates the evolution of a isodensity level in the orbital plane, for $b/L = 1.5$. The satellite is moving along the trajectory given by equation (44) and the displacement vector is computed for $GM/bV^2 = 0.01$. The satellite along its trail rises a tide and a bulge forms which lag behind.

In Figure 4, we draw a sequence of contour levels describing the extent of the deformation when the satellite hits the contour itself: here, $b/L_c = 0.5$ and $GM/bV^2 = 0.01$. The sharp decrement in the density behind M represents the region which is adjacent to the overdensity responsible to the frictional drag.

6.4 *Tidal force in shortlived distant encounters*

At high relative speeds ($V \gg < v^2 >^{1/2}$) and large impact parameters ($b \gg < r^2 >^{1/2}$, the distortion induced on the stellar system acts back to cause tidal drag. By expanding equation [43] in multipoles, we find, to leading order,

$$\mathbf{F}_\Delta = \mathbf{F}_T = [GM]^2 Nm \frac{< r^2 >}{V^2} \left(5(\mathbf{R} \cdot \mathbf{n}) \frac{\mathbf{R}}{R^7} - 2 \frac{\mathbf{n}}{R^5} \right) \tag{52}$$

resulting in a total energy loss ΔE which coincides with that of Spitzer, once equation (44) is adopted for the motion of the satellite. The force (52) represents the effect of the global tides on the satellite and has a component along $\mathbf{V}$ and a component along $\mathbf{R}$, due to the nonuniformities excited by the tidal interaction. The radial component is not conservative, i.e., it can not be written in terms of an effective potential but affects the energy balance equation.

In a flyby, the extent of the distortion increases with time. The bulge becomes pronounced after the satellite has reached the distance of closet approach and attains its maximum at a later time $t \sim 1/\omega$; the stars will then adjust into a new equilibrium. The results thus seem to indicate that there is a *time delay* in between the moment of closet approach and the moment of maximum deformation of the bulge. The magnitude of the distortion D can be computed using equation (39) and is approximately $(GM/V^2)(D/2L) \sim (V/ < v^2 >^{1/2})(\epsilon/b)(b/L - 1)^{-1/2}$. where we indicate with $2L$ the dimension of the galaxy.

To gain insight into the new result we need to explore further the role played by the tidal force. Can the global tidal distortion depicted in Figures (4) and (5) induce a drag on M , i.e., an effective energy and angular momentum loss for the satellite ? How does this loss compare in magnitude to the energy loss by friction ?

We thus need to estimate the total energy change (resulting from the friction and tidal components) along the satellite motion. The calculation, outlined in Appendix C, shows that $\Delta E < 0$ for any arbitrary stellar distribution, implying that orbital energy is transferred to the stars, resulting in an effective drag for the satellite. A *conservative* term from the tidal field is found which does not contribute to ΔE. The total energy loss for M decomposes into two terms

$$\Delta E = \Delta E_{+} + \Delta E_{*}. \tag{53}$$

The first reads

$$\Delta E_{+} = -\frac{[GM]^2}{V} Nm \int_{-\infty}^{+\infty} dt \frac{1}{R^2(t)} \int_{\mathcal{D}} d_3\mathbf{r}\ n_0(r) \frac{\mathbf{R}(t)\cdot\mathbf{r}}{|\mathbf{R}(t)-\mathbf{r}|^3}, \tag{54}$$

where the domain $\mathcal{D}$ is limited to those stellar encounters having $|\mathbf{R}(t)-\mathbf{r}| > \epsilon$ (we here restore the notation indicating the dependence of $\mathbf{R}$ on time t for seek of clearness). The second is

$$\Delta E_{*} = -[GM]^2 Nm \frac{b^2}{2} \left[\int_{-\infty}^{+\infty} dt \frac{\Omega_t}{R^3(t)}\right] \left[\int_{-\infty}^{+\infty} dt \frac{\bar{\Omega}_t}{R^3(t)}\right] \tag{55}$$

where Ω_t is a function of the actual position of the satellite

$$\Omega_t(R) \equiv \int_{r'<R(t)} d_3\mathbf{r}' n_0(r'); \tag{56}$$

Ω_t is proportional to the stellar mass comprised within a sphere of radius $R(t)$, and $\bar{\Omega} = 1 - \Omega$. ΔE_{*} gives a nonvanishing contribution to the energy loss only if the satellite in its past traveled inside the stellar background.

Restricting the analysis to the case of a finite size cloud of radius L, we are able to show that for $b/L > 1$ tidal drag leads to an energy loss given by equation (54)

$$\Delta E = -8\pi \frac{[GM]^2}{V^2} \rho_0 \left[L - \frac{L^3}{3b^2} - (b^2 - L^2)^{1/2} \sin^{-1}\frac{L}{b}\right] \tag{57}$$

In a close encounter (i.e., with $b < L$) the frictional force becomes important and the total energy loss can be written as sum of two contributions:

$$\begin{aligned}\Delta E_{+} = &-4\pi \frac{[GM]^2}{3V^2} \rho_0 \\ &\left[-\frac{2L^3}{b^2} + \frac{L^2 L_M}{b^2} + 6L - 7L_M - 3L_M \ln\frac{2L + L_M}{L_M} + 3L_M \ln L_M/\epsilon\right]\end{aligned} \tag{58}$$

and

$$\Delta E_{*} = -\frac{4\pi}{3} \frac{[GM]^2}{V^2} \rho_0 \frac{L_M^3}{4b^2} \left[1 - \frac{L_M^3}{8L^3}\right] \tag{59}$$

The last term in the square bracket of equation (58) represents the energy loss by dynamical friction (from eq. [47] and [49]): here we show that the relevant scale in the Coulomb logarithm is

$$b_{max} = L_M = 2(L^2 - b^2)^{1/2}, \tag{60}$$

the effective length traveled by satellite during its high speed encounter, within the galaxy. This is the first analytic self-consistent derivation of the maximum impact parameter b_{max}. The derivation of equations (57-59) is outlined in Appendix C.

Figure 5 depicts the relative contributions to the total energy loss. When $b > L$ the only energy loss is induces by the tidal response and is given by equation (57) (solid line). For $b < L$, the dot-dashed line refers to the contribution resulting solely from "friction" (from eq.[42]). The dashed line denotes the contribution from equation (59), while solid line gives the total energy loss. Clearly, in equation (58) the additional terms resulting from the tidal response tend to reduce the contribution resulting from friction. This goes in the direction indicated by numerical findings of Hernquist & Weinberg (1989; despite the misuse of the definition of the self-gravity of the response) and Zaritsky & White (1988). The decrement increases with decreasing b/L, as illustrated in Figure 5.

CONCLUSIONS

We have derived a formalism, within the linear response theory, for the analysis of the interaction of a massive particle (the perturber) with a spherical galaxy whose equilibrium is described by a one-particle distribution function.

We applied the perturbative technique to the case of a satellite moving at high speed across a stellar system. We found a natural decomposition of the back-reaction force into two components: a global component resulting from the "tidal" interaction and a component that is related to dynamical friction.

The results are relevant only to the description of shortlived encounters since, in the application, we neglected the self-gravity of the stellar background. In paper II (Colpi 1997) we will explore the accretion of a satellite orbiting around a spherical companion galaxy. In a consistent manner we will account for the self-gravity of stars and for the actual dynamics of the satellite. We will present a semianalytical method for the description of the process of tidal capture of a satellite, and for the analysis of the evolution of a " binary" with the aim at exploring the nature of the gravitational interaction between the two systems. Irreversible processes can cause orbital decay by exciting stars in "near" resonance with the orbit of the satellite; this phase precedes the final plunge in. The drag experienced by a satellite moving along an arbitrary orbit inside the galaxy will be the next problem that we wish to consider.

During final preparation of this work, Nelson & Tremaine (1997) submitted a paper where a formal study of the linear response theory is presented. The authors derive properties of the response operator in inhomogeneous dynamical systems and express the coefficient of dynamical friction in terms of the equilibrium fluctuations, analogously to our equation (16). Their theory is developed in an erbitrary inertial frame. We have instead chosen to describe tha process of irreversible energy transfer between the two components in a frame comoving with the center of mass of the galaxy (eq.[32]). As previously discussed, we expect that our formalism is preferable when an effective one–particle dynamics is adopted for the description of the unperturbed stellar motion because it automatically takes into account for the conservation of linear momentum in the scattering process of the galaxy whit the satellite.

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APPENDIX A

1. Total energy loss

In this Appendix we sketch the derivation of equation (23) for the energy loss experienced by the satellite

during the encounter, using equation (19):

$$\Delta E = \int_{-\infty}^{+\infty} dt \int_{-\infty}^{t} ds \int d_3\mathbf{r}\, d_3\mathbf{v}\ f^{\rm op} v_s^b T^{ba}\, V_t^a \tag{A1}$$

where index t (or s) denote that the variable is evaluated at time t (or s). The domain of integration over time s and t can be interchanged to give, in an equivalent form

$$\Delta E = \int_{-\infty}^{+\infty} ds \int_{s}^{+\infty} dt \int d_3\mathbf{r}\, d_3\mathbf{v}\ f^{\rm op} v_s^b T^{ba}\ (V^a + v^a - v^a)\,. \tag{A2}$$

Restoring the complete expression for the correlation tensor we have

$$\begin{aligned}\Delta E = &- [GM]^2 N m^2 \beta \int d_3\mathbf{r}\ d_3\mathbf{v}\ f^{\rm op} \\ &\int_{-\infty}^{+\infty} ds \int_{s}^{+\infty} dt v_s^b \frac{R_s^b - r_s^b}{|\mathbf{R}_s - \mathbf{r}_s|^3} \left[-\frac{d}{dt}\frac{1}{|\mathbf{R}_t - \mathbf{r}_t|} + v_t^a\ \frac{R_t^a - r_t^a}{|\mathbf{R}_t - \mathbf{r}_t|^3} \right] :\end{aligned} \tag{A3}$$

In the second term of equation (A3) the vectors $\mathbf{R}, \mathbf{r}$ and $\mathbf{v}$ are evaluated at time t.

Because of the isotropy of the unperturbed stellar system, the contribution to ΔE resulting from the first term in squared bracket,

$$-[GM]^2 N m^2 \beta \int d_3\mathbf{r}\ d_3\mathbf{v} f^{\rm op} \int_{-\infty}^{+\infty} ds\ v_s^b \frac{R_s^b - r_s^b}{|\mathbf{R}_s - \mathbf{r}_s|^4} \tag{A4}$$

vanishes identically. The second term gives instead equation (23) together with (24).

2. Impulse approximation

In the impulse approximation, the motion of the satellite and of the background stars is uniform. We can evaluate ΔE according to the simplifying assumption that $\mathbf{R}_t = \mathbf{b} + t\mathbf{V}$ and $\mathbf{r}_t = \mathbf{r} + t\mathbf{v}$, with $\mathbf{V}$ and $\mathbf{v}$ constant vectors and $\mathbf{b}\cdot\mathbf{V} = 0$. Defining $\mathbf{a} = \mathbf{b} - \mathbf{r}$ and $\mathbf{w} = \mathbf{V} - \mathbf{v}$, the scalar $\mathcal{J}$ is found to be

$$\mathcal{J} = \int_{-\infty}^{+\infty} dt\ \mathbf{v} \cdot \frac{\mathbf{a} + t\mathbf{w}}{|\mathbf{a} + t\mathbf{w}|^3} = 2\,\mathbf{v}\cdot\mathbf{B} \tag{A5}$$

with

$$\mathbf{B} \equiv \frac{\mathbf{a}_\perp}{a_\perp^2 w}. \tag{A6}$$

In equation (A6) $\mathbf{a}_\perp$ is the component of $\mathbf{a}$ in the orthogonal direction of vector $\mathbf{w}$ and is equal to $\mathbf{a}_\perp = \mathbf{a} - (\mathbf{a}\cdot\mathbf{w})\mathbf{w}/w^2$. The total energy loss thus reads

$$\Delta E = -2[GM]^2 N m^2 \beta \int d_3\mathbf{r}\ d_3\mathbf{v} f^{\rm op} (\mathbf{v}\cdot\mathbf{B})^2. \tag{A7}$$

The energy loss by recoil is recovered from equation (A7) in the limit $b \to \infty$ and $V \to \infty$ valid if the satellite moving at high speed maintains always at a distance from the stellar distribution. To first leading order, the vector $\mathbf{B}$ reduces to $\mathbf{B} = \mathbf{b}/(V\,b^2)$ yielding

$$\Delta E = -2[GM]^2 N m/(V^2\, b^2). \tag{A8}$$

(We recall that $\int d_3\mathbf{v} f^{\rm op}(v^a)^2 = \sigma^2 n_0(r)$.)

Instead, in the limit $V \to \infty$ but r/b finite,

$$\mathbf{B} = \frac{\mathbf{a} + (\mathbf{r}\cdot\mathbf{V})\mathbf{V}/V^2}{V\left[\mathbf{a} + (\mathbf{r}\cdot\mathbf{V})\mathbf{V}/V^2\right]}, \tag{A9}$$

and the energy loss equals

$$\Delta E = -2\frac{[GM]^2}{V^2} N m^2 \beta \sigma^2 \int d_3\mathbf{r}\, n_0(r) \frac{1}{[a^2 - (\mathbf{r}\cdot\mathbf{V})^2/V^2]}. \tag{A10}$$

Expanding equation (A10) to order $< r^2 > /b^4$ we recover Spitzer result (eq. [28]).

APPENDIX B

Wake and global tides

In this Appendix we summarize the main steps that lead to the decomposition of the force $\mathbf{F}_\Delta$ in terms of $\mathbf{F}_{DF}$ and $\mathbf{F}_\Delta$. Both components are derived to order $O(1/V^2)$.

The density response Δn is calculated from equation (36), in the high speed limit, according to equations (34) and (35). The force term (the one in squares brackets) and the density field $n_0(|\mathbf{r}-(t-s)\mathbf{v}|)$ are expanded, in equation (36), to first order in the velocity $\mathbf{v}$, contributing to order GM/V^2.

Let us denote with $\Delta_1 n$ the distortion resulting from the expansion of the force term, and with $\Delta_2 n$ the one from the expansion of the unperturbed density n_0. The global distortion will therefore be

$$\Delta n = \Delta_1 n + \Delta_2 n \tag{B1}$$

For ease of analysis, we introduce the vector

$$\mathbf{X} \equiv \mathbf{R} - \mathbf{r} - (t-s)\mathbf{V} \tag{B2}$$

so that

$$\frac{X^a + (t-s)v^a}{|\mathbf{X} + (t-s)\mathbf{v}|^3} = \frac{X^a}{X^3} + (t-s)v^b \frac{\partial}{\partial X^b}\frac{X^a}{|\mathbf{X}|^3} + O(v^2) \tag{B3}$$

to first order in $\mathbf{v}$ ($\mathbf{X}$ has no relation with the one introduced in §4.1).

According to equation (36)

$$\begin{aligned}\Delta_1 n =& GMm\beta\left(\frac{m\beta}{2\pi}\right)^{3/2}\int_{-\infty}^{t} ds \int d_3\mathbf{v}\; n_0(r) e^{-\frac{1}{2}\beta m v^2} \\ &\left\{v^a\left[\frac{X^a}{X^3} - \frac{R^a - (t-s)V^a}{|\mathbf{R} - (t-s)\mathbf{V}|^3}\int_{r'<d} d_3\mathbf{r}'\; n_0(r')\right] + (t-s)\, v^a\, v^b \frac{\partial}{\partial X^b}\frac{X^a}{X^3}\right\}\end{aligned} \tag{B4}$$

where d simply denotes $|\mathbf{R} - (t-s)\mathbf{V}|$. The term in squared braked vanishes identically because of isotropy. Only the term $\propto v^a\, v^b$ gives a contribution to the distortion. Since for a Maxwellian distribution function

$$\int d_3\mathbf{v}\; e^{-\frac{1}{2}\beta m v^2}\; v^a\, v^b = \frac{\pi^{3/2}}{2}\left(\frac{1}{2}m\beta\right)^{-5/2}\delta^{ab} \tag{B5}$$

we finally have

$$\Delta_1 n(\mathbf{r}) = GMn_0(r) \int_{-\infty}^{t} ds\; (t-s)\nabla_X \cdot \left[\frac{\mathbf{X}}{|\mathbf{X}|^3}\right]. \tag{B6}$$

Using the divergence theorem we have

$$\Delta_1 n(\mathbf{r}) = 4\pi\, GMn_0(r) \int_{-\infty}^{t} ds\ (t-s)\delta_3(\mathbf{X}) \tag{B7}$$

which is equivalent to equation (37).

The global distortion in the density field is derived expanding n_0 to first order in $\mathbf{v}$; using Gauss theorem we derive the following expression

$$\begin{aligned}\Delta_2 n(\mathbf{r}) =& GMm\beta \left(\frac{m\beta}{2\pi}\right)^{3/2} \left(-\frac{\partial n_0}{\partial r^b}\right) \int_{-\infty}^{t} ds\ (t-s) \int d_3\mathbf{v} e^{-\frac{1}{2}\beta m v^2} \\ & v^a\, v^b \left[\frac{X^a}{X^3} - \frac{R^a-(t-s)V^a}{|\mathbf{R}-(t-s)\mathbf{V}|^3}\int_{r'<d} d_3\mathbf{r}'\ n_0(r')\right]\end{aligned} \tag{B8}$$

Using equation (B5) and introducing the velocity vector $\mathbf{n} = \mathbf{V}/V$, we find that the density response reads

$$\begin{aligned}\Delta_2 n(\mathbf{r}) =& -\frac{GM}{V^2}\frac{\partial n}{\partial r^a} \\ & \int_0^{+\infty} dx\ x\ \left[\frac{R^a-\mathbf{r}-n^a x}{|\mathbf{R}-\mathbf{r}-\mathbf{n}x|^3} - \frac{R^a-n^a x}{|\mathbf{R}-\mathbf{n}x|^3} + \frac{R^a-n^a x}{|\mathbf{R}-\mathbf{r}-\mathbf{n}x|^3}\bar{\Omega}(|\mathbf{R}-\mathbf{n}x|)\right]\end{aligned} \tag{B9}$$

where $\bar{\Omega}$ is defined by equation (41). Equation provides the expression of the displacement vector in integral form and after lengthy calculations one recovers equation (39) for $\mathbf{D}$.

APPENDIX C

1. *The total energy loss*

In this Appendix we estimate the extent of the energy loss resulting from friction and from the tides excited by the satellite during its passage:

$$\Delta E = \int_{-\infty}^{+\infty} dt\ \mathbf{V}\cdot\mathbf{F}_\Delta \tag{C1}$$

Within TLR, the macroscopic force on M is found to be

$$\mathbf{F}_\Delta = -GMNm \int d_3\mathbf{r}[\Delta_1 n(\mathbf{r}) + \Delta_2 n(\mathbf{r})]\,\frac{\mathbf{R}-\mathbf{r}}{|\mathbf{R}-\mathbf{r}|^3} \tag{C2}$$

with $\Delta_1 n$ and $\Delta_2 n$ given by equations (37) and (38). The contribution coming from $\Delta_1 n$ directly gives:

$$\mathbf{F}_{\Delta_1} = -\frac{4\pi(GM)^2 Nm}{V^2}\int_0^\infty d\tau \frac{n_0(|\mathbf{R}-\mathbf{V}\tau|)}{\tau} \tag{C3}$$

while the term containing $\Delta_2 n$ in eq. (C2) can be written as the sum of three terms:

$$\begin{aligned}\int d_3\mathbf{r}\Delta_2 n(\mathbf{r})\,\frac{\mathbf{R}-\mathbf{r}}{|\mathbf{R}-\mathbf{r}|^3} =& -\frac{GM}{V^2}\int d_3\mathbf{r} \\ \left\{\nabla\cdot\left[\mathbf{D}_{\mathbf{R}-\mathbf{r}} n_0(r)\frac{\mathbf{R}-\mathbf{r}}{|\mathbf{R}-\mathbf{r}|^3}\right] - n_0(r)\right.&\left.\frac{\mathbf{R}-\mathbf{r}}{|\mathbf{R}-\mathbf{r}|^3}\nabla\cdot\mathbf{D}_{\mathbf{R}-\mathbf{r}} - n_0(r)\mathbf{D}_{\mathbf{R}-\mathbf{r}}\cdot\nabla\frac{\mathbf{R}-\mathbf{r}}{|\mathbf{R}-\mathbf{r}|^3}\right\}\end{aligned} \tag{C4}$$

where $\mathbf{D}_{\mathbf{R}-\mathbf{r}}$ is given by equation (39). The first term vanishes due to the divergence theorem, the second exactly cancels the Chandrasekhar term (C3) leaving only the third contribution which contains both frictional and tidal effects to give:

$$\mathbf{F}^a_\Delta = \frac{[GM]^2}{V^2} Nm \int d_3\mathbf{r} n_0(r) D^b_{(\mathbf{R}-\mathbf{r})} \ \nabla_{\mathbf{R}} \frac{R^b - r^b}{|\mathbf{R}-\mathbf{r}|^3}. \qquad (C5)$$

The force (C5) can be decomposed into the sum of a conservative contribution plus a component leading to dissipation of energy:

$$\begin{aligned}\mathbf{F}_\Delta =& \nabla_{\mathbf{R}} \left[\frac{[GM]^2}{V^2} Nm \int d_3\mathbf{r}\, n_0(r) \mathbf{D}_{(\mathbf{R}-\mathbf{r})} \cdot \frac{(\mathbf{R}-\mathbf{r})}{|\mathbf{R}-\mathbf{r}|^3} \right] \\ & - \frac{[GM]^2}{V^2} Nm \int_{\mathcal{D}} d_3\mathbf{r}\, n_0(r) \frac{R^b - r^b}{|\mathbf{R}-\mathbf{r}|^3} \nabla_{\mathbf{R}}\, D^b_{(\mathbf{R}-\mathbf{r})}. \end{aligned} \qquad (C6)$$

For ease of analysis we introduce the vector

$$\xi = \mathbf{R} - \mathbf{r} \qquad (C7)$$

expressing the relative distance between the satellite and the stars of the background. In the dissipative component the spatial integration is then limited to a domain $\mathcal{D}$ satisfying the condition $\xi > \epsilon$ (see eq. [27]).

The vector $\mathbf{D}$

$$D^b_\xi = \left[n^b \ln(\xi - \xi \cdot \mathbf{n}) + \frac{\xi^b - n^b(\xi \cdot \mathbf{n})}{\xi - \xi \cdot \mathbf{n}} \right] - \left[n^b \ln(R - \mathbf{R} \cdot \mathbf{n}) + \frac{R^b - n^b(\mathbf{R} \cdot \mathbf{n})}{R - \mathbf{R} \cdot \mathbf{n}} \right] + S^b \qquad (C8)$$

can be schematically written in the form

$$D^b_\xi = [\mathbf{n}, \xi]^b - [\mathbf{n}, \mathbf{R}]^b + S^b \qquad (C9)$$

where $\mathbf{S}$ is defined by equation (40).

In order to calculate the total energy loss we need to evaluate explicitly the component of the force along the velocity unit vector $\mathbf{n}$. Along the unit vector $\mathbf{n}$, the force is given

$$n^a\, F^a_\Delta = -\frac{[GM]^2}{V^2} Nm \int d_3\mathbf{r}\ n_0(r) n^a \frac{\xi^b}{\xi^3} \frac{\partial D^b}{\partial R^a} \qquad (C10)$$

One can prove that

$$n^a \frac{\xi^b}{\xi^3} \frac{\partial}{\partial \xi^a} [\mathbf{n}, \xi]^b = \frac{1}{\xi^3} \qquad (C11)$$

and

$$n^a \frac{\xi^b}{\xi^3} \frac{\partial}{\partial R^a} [\mathbf{n}, \mathbf{R}]^b = \frac{\xi^b}{\xi^3} \frac{R^b - n^b R}{R(R - \mathbf{n} \cdot \mathbf{R})} \qquad (C12)$$

Substituting (C11) and (C12) in (C10), and using Gauss theorem, we find

$$n^a\, F^a_\Delta = -\frac{[GM]^2}{V^2} Nm \left\{ -\frac{\Omega(R)}{R^3} + n^a \frac{\partial S^b}{\partial R^a} \frac{R^b}{R^3} \Omega(R) \right\} + \int d_3\mathbf{r}\ \frac{n_0(r)}{|\mathbf{R}-\mathbf{r}|^3}. \qquad (C13)$$

Noting that

$$\frac{\Omega(R)}{R^3} \equiv \frac{1}{R^3} \int_{r<R} d_3\mathbf{r}\ n_0(r) = \int d_3\mathbf{r} \frac{n_0(r)}{|\mathbf{R}-\mathbf{r}|^3} - \frac{1}{R^2} \int d_3\mathbf{r}\ n_0(r) \frac{\mathbf{R} \cdot \mathbf{r}}{|\mathbf{R}-\mathbf{r}|^3} \qquad (C14)$$

we find that ΔE splits into two terms $\Delta E_+ + \Delta E_*$ given by

$$\Delta E_+ = -\frac{[GM]^2}{V} Nm \int_{-\infty}^{+\infty} dt \frac{1}{R^2} \int d_3\mathbf{r}\ n_0(r) \frac{\mathbf{R} \cdot \mathbf{r}}{|\mathbf{R}-\mathbf{r}|^3} \qquad (C15)$$

and

$$\Delta E_* = -\frac{[GM]^2}{V} Nm \int_{-\infty}^{+\infty} dt \; n^a \; \frac{\partial S^b}{\partial R^a} \frac{R^b}{R^3} \Omega(R). \tag{C16}$$

Both terms are negative as shown below.

As regard to the first term ΔE_+ (C15), owing to the isotropy of the background, we can arbitrarily select a direction for $\mathbf{R}$ (along z axis) so that the resulting volume integral

$$\int d_3\mathbf{r} \; n_0(r) \frac{R\,z}{(R^2+r^2)^{3/2}} [1 - 2Rz/(R^2+r^2)]^{-3/2} \tag{C17}$$

has a kernel of the form $(1-z)^{-3/2} = [1 + 3z/2 - 15z^2/8 + \; O(z^3)]$ when expanded in series. Since only the terms with odd powers give a nonvanishing contribution, the volume integral has definite positive sign, resulting in a net energy loss. Also the term in equation (C16) can be suitably written in a form involving only positive functions.

We know evaluate ΔE_*. If we explicitly insert the expression of $\mathbf{S}$ as given by equations (40) and (41) into equation (C16) we have that the integral

$$I \equiv \frac{\Omega(R)}{R^3} n^a R^b \frac{\partial}{\partial R^a} \int_0^{+\infty} dx \; x \; \frac{R^b - n^b x}{|\mathbf{R} - \mathbf{n}\,x|^3} \bar{\Omega}(|\mathbf{R} - \mathbf{n}\,x|) \tag{C18}$$

is equal to

$$I = -\frac{\Omega(R)}{R^3} R^b \int_0^{+\infty} dx \; x \frac{\partial}{\partial x} \left[\frac{R^b - n^b x}{|\mathbf{R} - \mathbf{n}\,x|^3} \bar{\Omega}(|\mathbf{R} - \mathbf{n}\,x|) \right] \tag{C19}$$

giving a contribution to the energy loss

$$\Delta E_* = -[GM]^2 Nm \int_{-\infty}^{+\infty} dt \frac{\Omega(R_t)}{R_t^3} \int_0^{\infty} ds \frac{R_t^2 - \mathbf{R}_t \cdot \mathbf{V} s}{|\mathbf{R}_t - \mathbf{V}\,s|^3} \bar{\Omega}(|\mathbf{R}_t - \mathbf{V}\,s|) \tag{C20}$$

where we recovered the dependence upon t of $\mathbf{R}_t$, denoting the satellite position at current time t.

Since, in a high speed encounter, the satellite moves along a trajectory that can be approximated as linear, we can evaluate explicitly the integral over the variable s using equation (44). Noting that the following equality holds

$$\int_{-\infty}^{+\infty} dt \frac{\bar{\Omega}(R_t)}{R_t^3} = \int_{-\infty}^{+\infty} dt \frac{[1 - \Omega(R_t)]}{R_t^3} = \frac{2}{b^2 V} - \int_{-\infty}^{+\infty} dt \frac{\Omega(R_t)}{R_t^3} \tag{C21}$$

after some calculation we find that

$$\Delta E_* = \frac{[GM]^2}{V} Nm \left[\frac{b^2 V}{2} \left(\int_{-\infty}^{+\infty} dt \frac{\bar{\Omega}}{R_t^3} \right)^2 - \int_{-\infty}^{+\infty} dt \frac{\bar{\Omega}}{R_t^3} \right]. \tag{C22}$$

Using equation (C21) we recover eq.(55).

2. *Energy loss in a uniform cloud*

Here we provide a quantitative estimate of the total energy loss and determine the relative importance of the tidal and frictional contributions to the process of dissipation. We consider the case of a uniform cloud of mass density $\rho_0 = Nmn_0$ and radius L.

CASE A: If the satellite travels outside the spherical cloud the only contribution to the energy loss comes from the tidal field and is given by

$$\Delta E_+ = -\frac{[GM]^2}{V} \rho_0 \int_{-\infty}^{+\infty} dt \left[-\frac{4\pi}{3} \frac{L^3}{R_t^3} - 4\pi \frac{L}{R_t} + 2\pi \ln \frac{R_t + L}{R_t - L} \right] \tag{C23}$$

Since $R_t = (b^2 + (Vt)^2)^{1/2}$, the integral over time t can be evaluated analytically yielding

$$\Delta E_+ = -8\pi \frac{[GM]^2}{V^2} \rho_0 \left[L - \frac{L^3}{3b^2} - (b^2 - L^2)^{1/2} \sin^{-1} \frac{L}{b} \right] \tag{C24}$$

where b is the impact parameter ($b > L$). In the limit $b \gg L$ equation (C24) reduces to (28).

CASE B: If during the shortlived encounter the satellite enters the spherical cloud ($b < L$), the total energy loss is given by equation (54) plus (55). Notice that the satellite travels a distance $L_M = 2(L^2 - b^2)^{1/2}$, within the cloud, over a time $\tau_e = L_M/V$. In this case

$$\Delta E_+ = -\frac{[GM]^2}{V^2} \rho_0 \left[\int_{-L_M/2}^{L_M/2} d(Vt)\,(I_< + I_>) + 2 \int_{-\infty}^{-L_M/2} d(Vt) I_o \right] \tag{C25}$$

where $I_<$ $(I_>)$ is the volume integral resulting from the region of the cloud with $[0 < r < R - \epsilon]$ ($[L > r > R + \epsilon]$ respectively):

$$I_< + I_> = -\frac{16\pi}{3} + 2\pi \ln \frac{L^2 - R_t^2}{\epsilon^2} \tag{C26}$$

while

$$I_o = -\frac{4\pi}{3} \frac{L^3}{R^3} - 4\pi \frac{L}{R} + 2\pi \ln \frac{R + L}{R - L} \tag{C27}$$

The time integrals can then be carried out analytically:

$$\begin{aligned} \Delta E_+ = &- 4\pi \frac{[GM]^2}{3V^2} \rho_0 \\ &\left[-\frac{2L^3}{b^2} + \frac{L^2 L_M}{b^2} + 6L - 7L_M - 3L_M \ln \frac{2L + L_M}{L_M} - 3L_M \ln \frac{\epsilon}{L_M} \right] \end{aligned} \tag{C28}$$

Similarity, we can carry out the integration of equation (55) involving the functions Ω and $\bar{\Omega}$. The calculation is simple and lead to the following expression for

$$\Delta E_* = -\frac{4\pi}{3} \frac{[GM]^2}{V^2} \rho_0 \frac{L_M^3}{4b^2} \left[1 - \frac{L_M^3}{8L^3} \right] \tag{C29}$$

where $L_M = 2\sqrt{L^2 - b^2}$ is the length of the portion of trajectory within the cloud.

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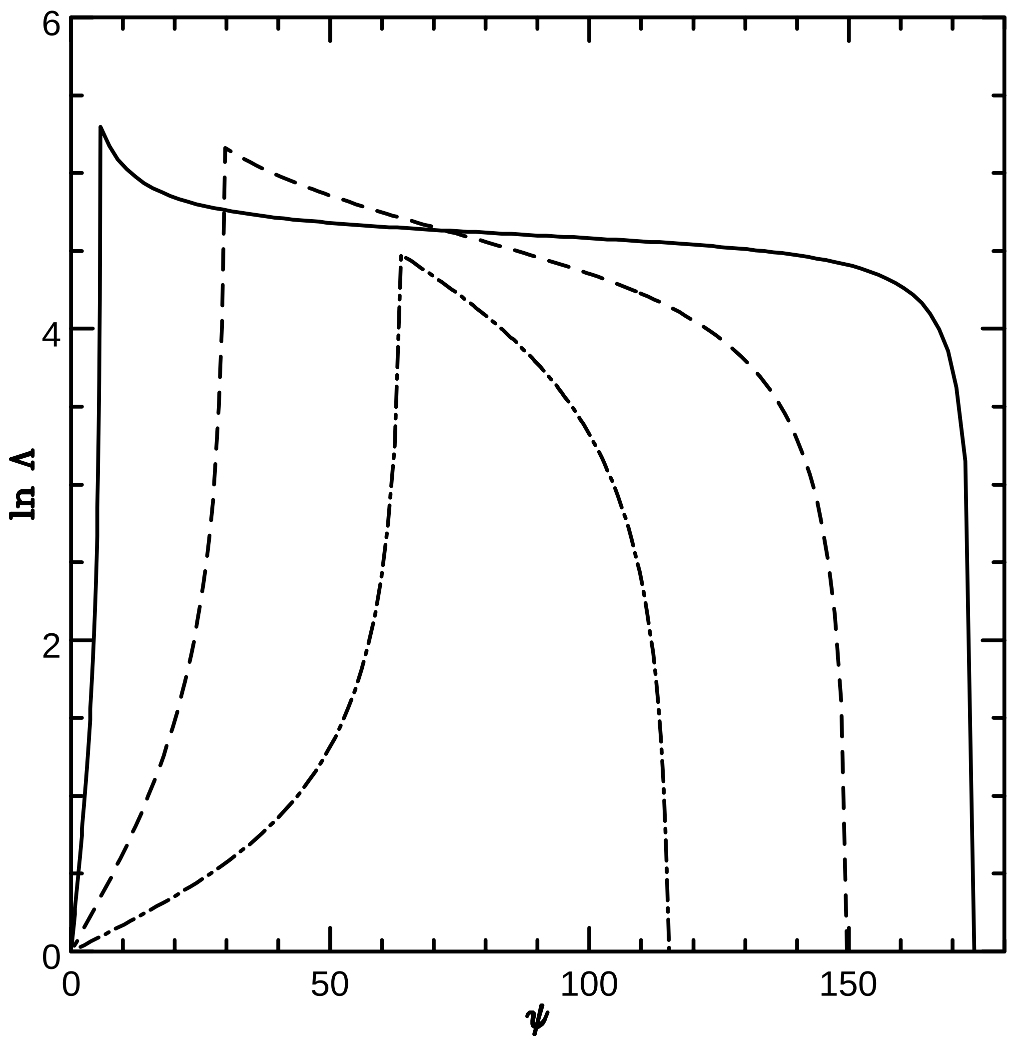
Figure Caption

Figure 1: Effective Coulomb logarithm versus phase ψ for a uniform finite size cloud *Solid* line is for $b = 0.1$; *dashed* line for $b = 0.5$ and *dashed-dotted* line for $b = 0.9$.
Figure 2: Effective Coulomb logarithm (*solid* line; eq.[51]) against phase ψ for $b = 0.9$ (right panel) and for $b = 0.5$ (left panel); $\epsilon/L = 0.01$. The spherical stellar background has density profile given by eq. [50] with $\gamma = 3/2$. *Dashed* line gives the comparison with the case of a uniform density cloud.
Figure 3 Isodensity contours, in the orbital plane (x, y) for $b/L = 1.5$: *Square* denotes the position of the satellite moving along the x axis, from left to right. The phase ψ decreases as time elapses and the four panels refer to $\psi = 120, 90, 60, 45$ degrees respectively.
Figure 4 Isodensity contours, in the orbital plane (x, y) for $b/L = 0.5$: *Square* denotes the position of the satellite moving along the x axis, from left to right. The phase ψ decreases as time elapses and the four panels refer to $\psi = 157, 130, 50, 26$ degrees respectively.
Figure 5 Energy loss $-\Delta E$ in units of $[GM]^2\rho_0\, L/V^2$ as a function of b/L. The spherical system is uniform and has size L. For $b/L > 1$ only tides cause dissipation of energy (eq. [57]-*solid* line) For $b/L < 1$ the tidal contribution of equation (59) is indicated by the *dashed* and the *dot-dashed* line denotes the contribution from the logarithmic term of equation (58) with $\epsilon/L = 0.01$. The *solid* line gives the total energy loss.



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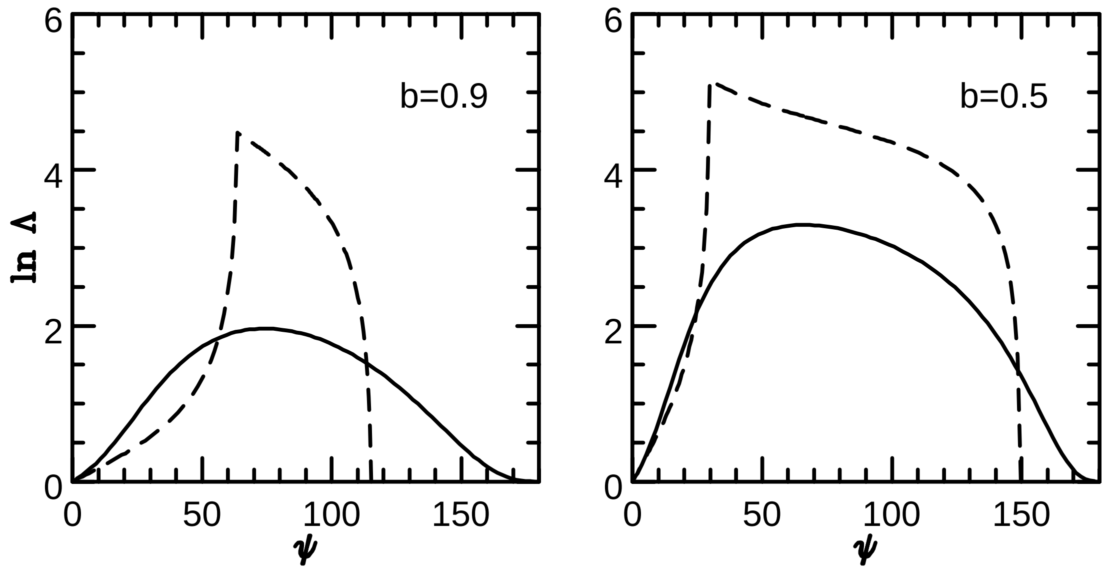

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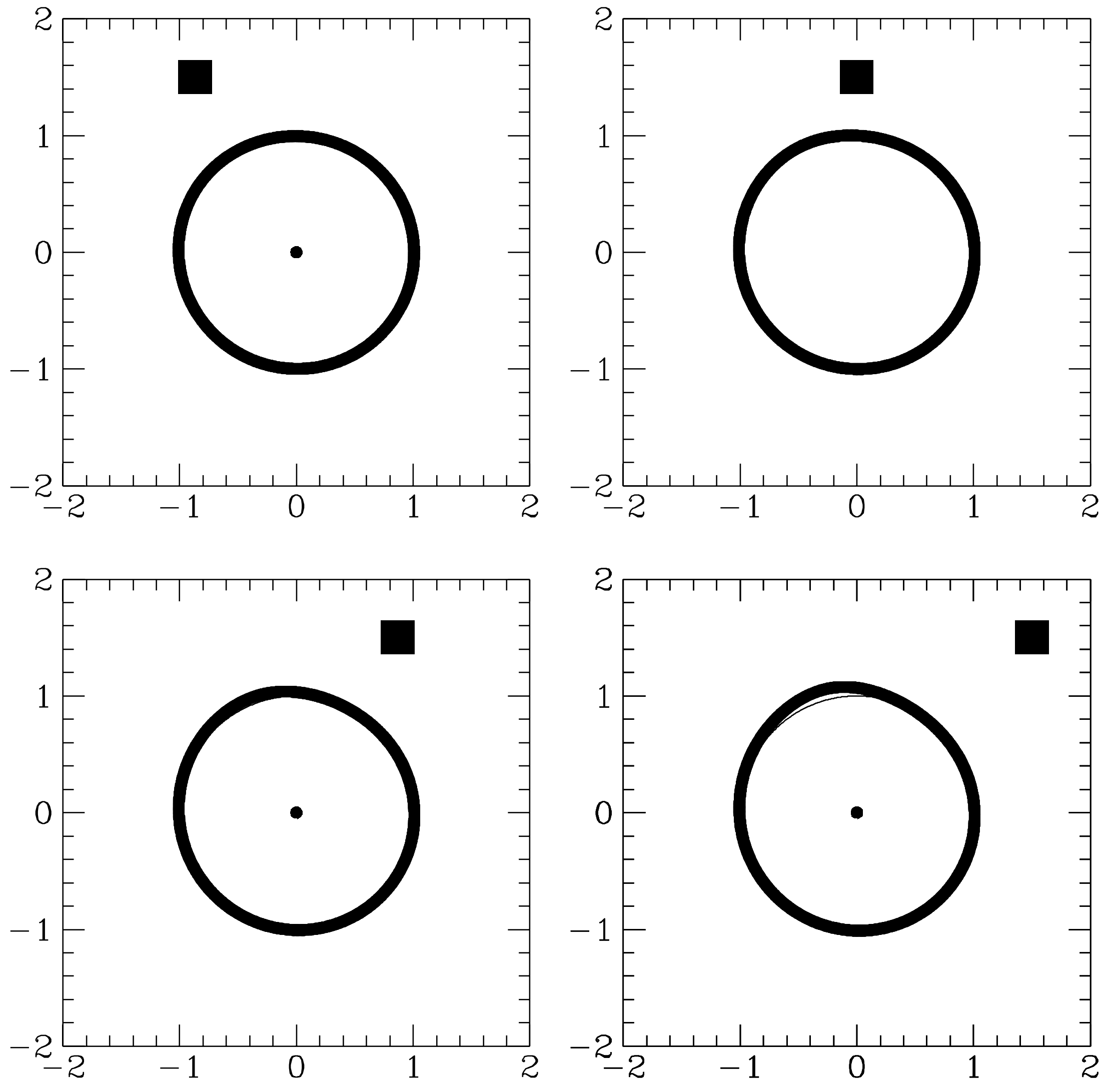

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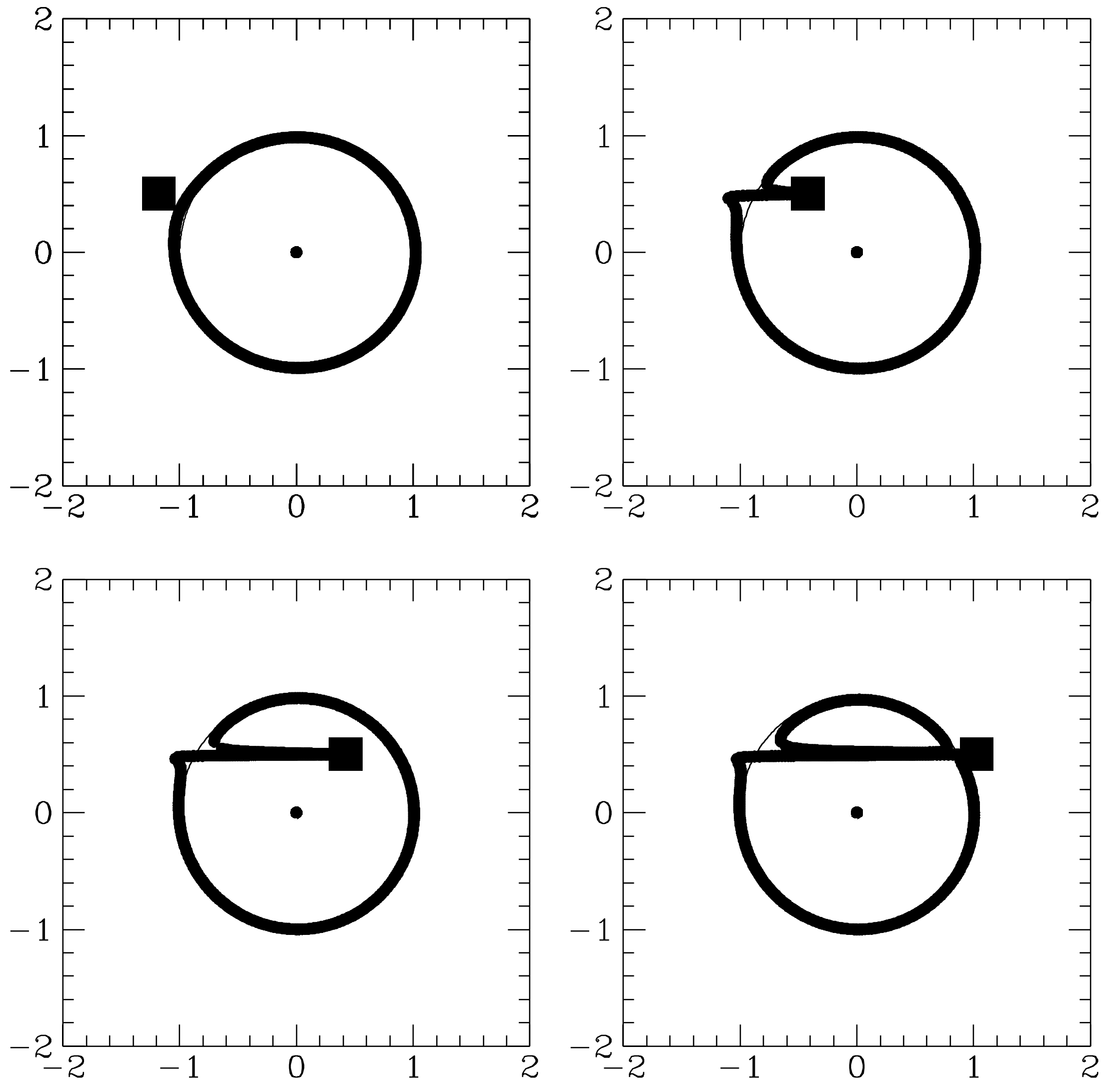

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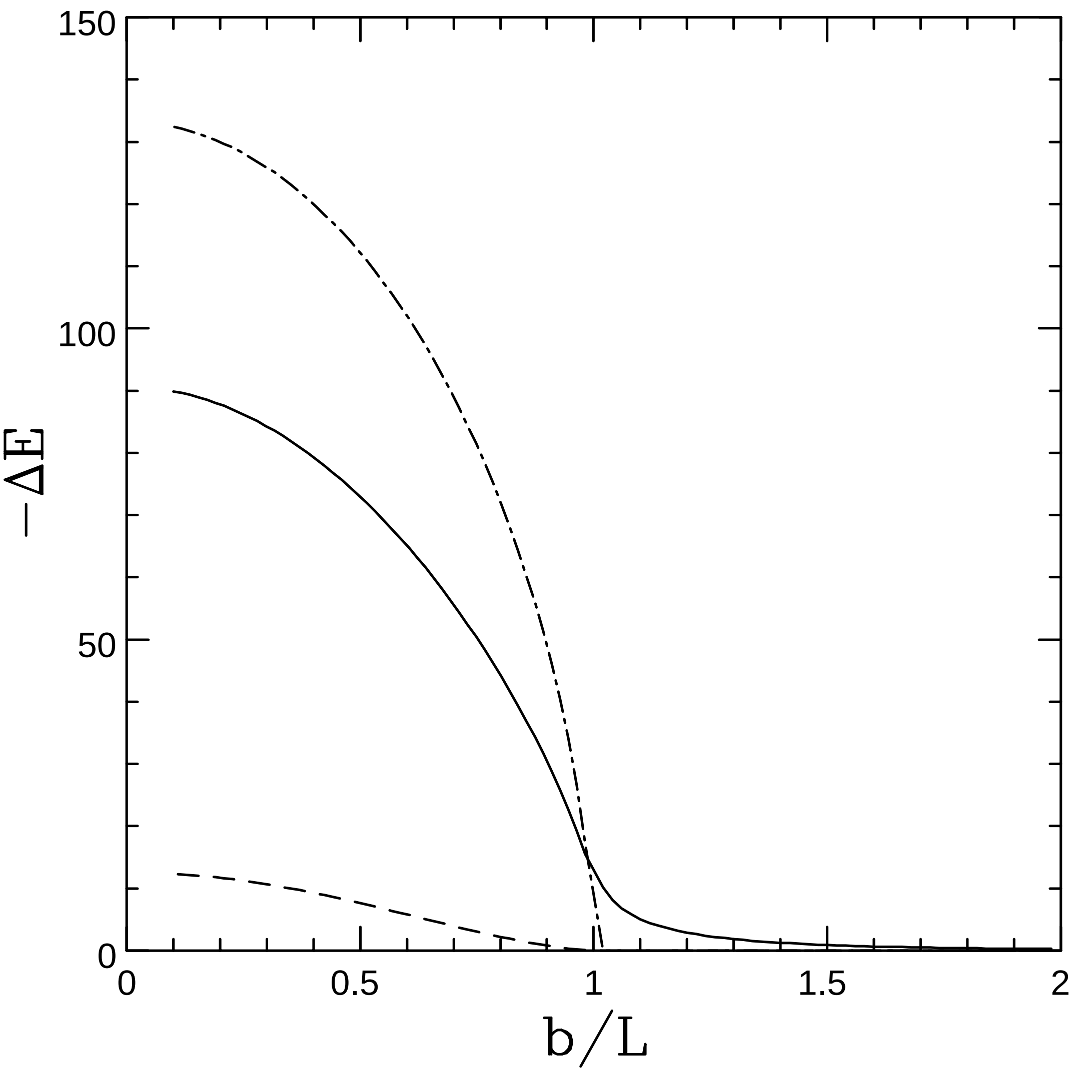

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$-\Delta E$
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